Mathematica 11.3 Integration Test Results

Test results for the 50 problems in "5.5.2 Inverse secant functions.m"

Problem 14: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcSec}\left[\frac{a}{x}\right]}{x^2}\,dx$$

Optimal (type 3, 31 leaves, 5 steps):

$$-\frac{\text{ArcCos}\left[\frac{x}{a}\right]}{x}+\frac{\text{ArcTanh}\left[\sqrt{1-\frac{x^2}{a^2}}\right]}{a}$$

Result (type 3, 93 leaves):

$$-\frac{\text{ArcSec}\left[\frac{a}{x}\right]}{x}+\frac{\sqrt{-1+\frac{a^2}{x^2}}\,x\left(-\text{Log}\left[1-\frac{a}{\sqrt{-1+\frac{a^2}{x^2}}\,x}\right]+\text{Log}\left[1+\frac{a}{\sqrt{-1+\frac{a^2}{x^2}}\,x}\right]\right)}{2\,a^2\sqrt{1-\frac{x^2}{a^2}}}$$

Problem 17: Result unnecessarily involves higher level functions.

$$\int \frac{\text{ArcSec}[a\,x^n]}{x}\,dx$$

Optimal (type 4, 69 leaves, 7 steps):

$$\frac{i\,\text{ArcSec}[a\,x^n]^2}{2\,n}-\frac{\text{ArcSec}[a\,x^n]\,\text{Log}\left[1+e^{2\,i\,\text{ArcSec}[a\,x^n]}\right]}{n}+\frac{i\,\text{PolyLog}\left[2,\,-e^{2\,i\,\text{ArcSec}[a\,x^n]}\right]}{2\,n}$$

Result (type 5, 60 leaves):

$$\frac{x^{-n}\,\text{HypergeometricPFQ}\left[\left\{\frac{1}{2},\frac{1}{2},\frac{1}{2}\right\},\left\{\frac{3}{2},\frac{3}{2}\right\},\frac{x^{-2n}}{a^2}\right]}{a\,n}+\left(\text{ArcSec}\left[a\,x^n\right]+\text{ArcSin}\left[\frac{x^{-n}}{a}\right]\right)\text{Log}[x]$$

Problem 22: Result more than twice size of optimal antiderivative.

$$\int \text{ArcSec}[a+b\,x]\,dx$$

Optimal (type 3, 37 leaves, 5 steps):

$$\frac{(a+b x) \operatorname{ArcSec}[a+b x]}{b}-\frac{\operatorname{ArcTanh}\left[\sqrt{1-\frac{1}{(a+b x)^2}}\right]}{b}$$

Result (type 3, 121 leaves):

$$x \operatorname{ArcSec}[a+b x]-\left((a+b x) \sqrt{\frac{-1+a^2+2 a b x+b^2 x^2}{(a+b x)^2}}\left(a \operatorname{ArcTan}\left[\frac{1}{\sqrt{-1+a^2+2 a b x+b^2 x^2}}\right]+\operatorname{Log}\left[a+b x+\sqrt{-1+a^2+2 a b x+b^2 x^2}\right]\right)\right) \Big/ \left(b \sqrt{-1+a^2+2 a b x+b^2 x^2}\right)$$

Problem 24: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{ArcSec}[a+b x]}{x^2} \, dx$$

Optimal (type 3, 70 leaves, 5 steps):

$$-\frac{b \operatorname{ArcSec}[a+b x]}{a}-\frac{\operatorname{ArcSec}[a+b x]}{x}+\frac{2 b \operatorname{ArcTan}\left[\frac{\sqrt{1+a} \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcSec}[a+b x]\right]}{\sqrt{1-a}}\right]}{a \sqrt{1-a^2}}$$

Result (type 3, 112 leaves):

$$-\frac{\operatorname{ArcSec}[a+b x]}{x}+\frac{b\left(\operatorname{ArcSin}\left[\frac{1}{a+b x}\right]-\frac{i \operatorname{Log}\left[\frac{2\left(\frac{i a\left(-1+a^2+a b x\right)}{\sqrt{1-a^2}}+a(a+b x) \sqrt{\frac{-1+a^2+2 a b x+b^2 x^2}{(a+b x)^2}}\right)}{b x}\right]}{\sqrt{1-a^2}}\right)}{a}$$

Problem 25: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{ArcSec}[a+b x]}{x^3} \, dx$$

Optimal (type 3, 125 leaves, 7 steps):

$$\frac{b(a+b x) \sqrt{1-\frac{1}{(a+b x)^2}}}{2 a\left(1-a^2\right) x}+\frac{b^2 \operatorname{ArcSec}[a+b x]}{2 a^2}-\frac{\operatorname{ArcSec}[a+b x]}{2 x^2}-\frac{\left(1-2 a^2\right) b^2 \operatorname{ArcTan}\left[\frac{\sqrt{1+a} \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcSec}[a+b x]\right]}{\sqrt{1-a}}\right]}{a^2\left(1-a^2\right)^{3/2}}$$

Result (type 3, 198 leaves):

$$-\frac{1}{2x^2}\left(\frac{b\,x\,(a+b\,x)\sqrt{\frac{-1+a^2+2\,a\,b\,x+b^2\,x^2}{(a+b\,x)^2}}}{a\,(-1+a^2)}+\text{ArcSec}[a+b\,x]+\frac{b^2\,x^2\,\text{ArcSin}\left[\frac{1}{a+b\,x}\right]}{a^2}+\frac{1}{a^2\,(1-a^2)^{3/2}}\right.$$
$$\left.i\,(-1+2\,a^2)\,b^2\,x^2\,\text{Log}\left[\frac{4\,(-1+a)\,a^2\,(1+a)\left(-\frac{i\,(-1+a^2+a\,b\,x)}{\sqrt{1-a^2}}-(a+b\,x)\sqrt{\frac{-1+a^2+2\,a\,b\,x+b^2\,x^2}{(a+b\,x)^2}}\right)}{(-1+2\,a^2)\,b^2\,x}\right]\right)$$

Problem 26: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\text{ArcSec}[a+b\,x]}{x^4}\,dx$$

Optimal (type 3, 181 leaves, 8 steps):

$$\frac{b\,(a+b\,x)\sqrt{1-\frac{1}{(a+b\,x)^2}}}{6\,a\,(1-a^2)\,x^2}-\frac{(2-5\,a^2)\,b^2\,(a+b\,x)\sqrt{1-\frac{1}{(a+b\,x)^2}}}{6\,a^2\,(1-a^2)^2\,x}-$$
$$\frac{b^3\,\text{ArcSec}[a+b\,x]}{3\,a^3}-\frac{\text{ArcSec}[a+b\,x]}{3\,x^3}+\frac{(2-5\,a^2+6\,a^4)\,b^3\,\text{ArcTan}\left[\frac{\sqrt{1+a}\,\text{Tan}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]}{\sqrt{1-a}}\right]}{3\,a^3\,(1-a^2)^{5/2}}$$

Result (type 3, 241 leaves):

$$\frac{1}{6}\left(-\frac{b\sqrt{\frac{-1+a^2+2\,a\,b\,x+b^2\,x^2}{(a+b\,x)^2}}\,(a^4+a\,b\,x-4\,a^3\,b\,x+2\,b^2\,x^2-a^2\,(1+5\,b^2\,x^2))}{a^2\,(-1+a^2)^2\,x^2}-\right.$$
$$\frac{2\,\text{ArcSec}[a+b\,x]}{x^3}+\frac{2\,b^3\,\text{ArcSin}\left[\frac{1}{a+b\,x}\right]}{a^3}-\frac{1}{a^3\,(1-a^2)^{5/2}}$$
$$\left.i\,(2-5\,a^2+6\,a^4)\,b^3\,\text{Log}\left[\frac{12\,a^3\,(-1+a^2)^2\left(\frac{i\,(-1+a^2+a\,b\,x)}{\sqrt{1-a^2}}+(a+b\,x)\sqrt{\frac{-1+a^2+2\,a\,b\,x+b^2\,x^2}{(a+b\,x)^2}}\right)}{(2-5\,a^2+6\,a^4)\,b^3\,x}\right]\right)$$

Problem 27: Result more than twice size of optimal antiderivative.

$$\int x^3\,\text{ArcSec}[a+b\,x]^2\,dx$$

Optimal (type 4, 381 leaves, 20 steps):

$$-\frac{a x}{b^3}+\frac{(a+b x)^2}{12 b^4}-\frac{(a+b x)\sqrt{1-\frac{1}{(a+b x)^2}}\,\text{ArcSec}[a+b x]}{3 b^4}-\frac{3 a^2 (a+b x)\sqrt{1-\frac{1}{(a+b x)^2}}\,\text{ArcSec}[a+b x]}{b^4}+\frac{a (a+b x)^2\sqrt{1-\frac{1}{(a+b x)^2}}\,\text{ArcSec}[a+b x]}{b^4}-\frac{(a+b x)^3\sqrt{1-\frac{1}{(a+b x)^2}}\,\text{ArcSec}[a+b x]}{6 b^4}-\frac{a^4\,\text{ArcSec}[a+b x]^2}{4 b^4}+\frac{1}{4} x^4\,\text{ArcSec}[a+b x]^2-\frac{2 i a\,\text{ArcSec}[a+b x]\,\text{ArcTan}\left[e^{i\,\text{ArcSec}[a+b x]}\right]}{b^4}-\frac{4 i a^3\,\text{ArcSec}[a+b x]\,\text{ArcTan}\left[e^{i\,\text{ArcSec}[a+b x]}\right]}{b^4}+\frac{\text{Log}[a+b x]}{3 b^4}+\frac{3 a^2\,\text{Log}[a+b x]}{b^4}+\frac{i a\,\text{PolyLog}\left[2,-i e^{i\,\text{ArcSec}[a+b x]}\right]}{b^4}+\frac{2 i a^3\,\text{PolyLog}\left[2,-i e^{i\,\text{ArcSec}[a+b x]}\right]}{b^4}-\frac{i a\,\text{PolyLog}\left[2,i e^{i\,\text{ArcSec}[a+b x]}\right]}{b^4}-\frac{2 i a^3\,\text{PolyLog}\left[2,i e^{i\,\text{ArcSec}[a+b x]}\right]}{b^4}$$

Result (type 4, 1141 leaves):

$$\frac{1}{b^4}\left(\frac{a b^3 x^3\left(2+\text{ArcSec}[a+b x]^2+2 a^2\,\text{ArcSec}[a+b x]^2\right)}{2 (a+b x)^3\left(-1+\frac{a}{a+b x}\right)^3}-\frac{\left(-\frac{1}{3}-3 a^2\right) b^3 x^3\,\text{Log}\left[\frac{1}{a+b x}\right]}{(a+b x)^3\left(-1+\frac{a}{a+b x}\right)^3}+\left(\left(-a-2 a^3\right) b^3 x^3\left(\left(\frac{\pi}{2}-\text{ArcSec}[a+b x]\right)\left(\text{Log}\left[1-e^{i\left(\frac{\pi}{2}-\text{ArcSec}[a+b x]\right)}\right]-\text{Log}\left[1+e^{i\left(\frac{\pi}{2}-\text{ArcSec}[a+b x]\right)}\right]\right)-\frac{1}{2}\pi\,\text{Log}\left[\text{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-\text{ArcSec}[a+b x]\right)\right]\right]+i\left(\text{PolyLog}\left[2,-e^{i\left(\frac{\pi}{2}-\text{ArcSec}[a+b x]\right)}\right]-\text{PolyLog}\left[2,e^{i\left(\frac{\pi}{2}-\text{ArcSec}[a+b x]\right)}\right]\right)\right)\right)\Big/\left((a+b x)^3\left(-1+\frac{a}{a+b x}\right)^3\right)-\left(b^3 x^3\,\text{ArcSec}[a+b x]^2\right)\Big/\left(16 (a+b x)^3\left(-1+\frac{a}{a+b x}\right)^3\left(\text{Cos}\left[\frac{1}{2}\text{ArcSec}[a+b x]\right]-\text{Sin}\left[\frac{1}{2}\text{ArcSec}[a+b x]\right]\right)^4\right)+\left(b^3 x^3\left(-2+2\,\text{ArcSec}[a+b x]-24 a\,\text{ArcSec}[a+b x]-3\,\text{ArcSec}[a+b x]^2+12 a\,\text{ArcSec}[a+b x]^2-36 a^2\,\text{ArcSec}[a+b x]^2\right)\right)\Big/\left(48 (a+b x)^3\left(-1+\frac{a}{a+b x}\right)^3\left(\text{Cos}\left[\frac{1}{2}\text{ArcSec}[a+b x]\right]-\text{Sin}\left[\frac{1}{2}\text{ArcSec}[a+b x]\right]\right)^2\right)-\left(b^3 x^3\,\text{ArcSec}[a+b x]^2\right)\Big/\right.$$

$$\left(16\,(a+b\,x)^3\left(-1+\frac{a}{a+b\,x}\right)^3\left(\text{Cos}\left[\frac{1}{2}\,\text{ArcSec}[a+b\,x]\right]+\text{Sin}\left[\frac{1}{2}\,\text{ArcSec}[a+b\,x]\right]\right)^4\right)+$$

$$\left(b^3\,x^3\left(-2-2\,\text{ArcSec}[a+b\,x]+24\,a\,\text{ArcSec}[a+b\,x]-3\,\text{ArcSec}[a+b\,x]^2+12\,a\,\text{ArcSec}[a+b\,x]^2-36\,a^2\,\text{ArcSec}[a+b\,x]^2\right)\right)\Big/$$

$$\left(48\,(a+b\,x)^3\left(-1+\frac{a}{a+b\,x}\right)^3\left(\text{Cos}\left[\frac{1}{2}\,\text{ArcSec}[a+b\,x]\right]+\text{Sin}\left[\frac{1}{2}\,\text{ArcSec}[a+b\,x]\right]\right)^2\right)+$$

$$\left(b^3\,x^3\left(\text{ArcSec}[a+b\,x]\,\text{Sin}\left[\frac{1}{2}\,\text{ArcSec}[a+b\,x]\right]-6\,a\,\text{ArcSec}[a+b\,x]^2\,\text{Sin}\left[\frac{1}{2}\,\text{ArcSec}[a+b\,x]\right]\right)\right)\Big/$$

$$\left(12\,(a+b\,x)^3\left(-1+\frac{a}{a+b\,x}\right)^3\left(\text{Cos}\left[\frac{1}{2}\,\text{ArcSec}[a+b\,x]\right]+\text{Sin}\left[\frac{1}{2}\,\text{ArcSec}[a+b\,x]\right]\right)^3\right)+$$

$$\left(b^3\,x^3\left(\text{ArcSec}[a+b\,x]\,\text{Sin}\left[\frac{1}{2}\,\text{ArcSec}[a+b\,x]\right]+6\,a\,\text{ArcSec}[a+b\,x]^2\,\text{Sin}\left[\frac{1}{2}\,\text{ArcSec}[a+b\,x]\right]\right)\right)\Big/$$

$$\left(12\,(a+b\,x)^3\left(-1+\frac{a}{a+b\,x}\right)^3\left(\text{Cos}\left[\frac{1}{2}\,\text{ArcSec}[a+b\,x]\right]-\text{Sin}\left[\frac{1}{2}\,\text{ArcSec}[a+b\,x]\right]\right)^3\right)+$$

$$\left(b^3\,x^3\left(-6\,a\,\text{Sin}\left[\frac{1}{2}\,\text{ArcSec}[a+b\,x]\right]+2\,\text{ArcSec}[a+b\,x]\,\text{Sin}\left[\frac{1}{2}\,\text{ArcSec}[a+b\,x]\right]+\right.\right.$$
$$18\,a^2\,\text{ArcSec}[a+b\,x]\,\text{Sin}\left[\frac{1}{2}\,\text{ArcSec}[a+b\,x]\right]-3\,a\,\text{ArcSec}[a+b\,x]^2\,\text{Sin}\left[\frac{1}{2}\,\text{ArcSec}[a+b\,x]\right]-$$
$$\left.\left.6\,a^3\,\text{ArcSec}[a+b\,x]^2\,\text{Sin}\left[\frac{1}{2}\,\text{ArcSec}[a+b\,x]\right]\right)\right)\Big/$$

$$\left(6\,(a+b\,x)^3\left(-1+\frac{a}{a+b\,x}\right)^3\left(\text{Cos}\left[\frac{1}{2}\,\text{ArcSec}[a+b\,x]\right]+\text{Sin}\left[\frac{1}{2}\,\text{ArcSec}[a+b\,x]\right]\right)\right)+$$

$$\left(b^3\,x^3\left(6\,a\,\text{Sin}\left[\frac{1}{2}\,\text{ArcSec}[a+b\,x]\right]+2\,\text{ArcSec}[a+b\,x]\,\text{Sin}\left[\frac{1}{2}\,\text{ArcSec}[a+b\,x]\right]+\right.\right.$$
$$18\,a^2\,\text{ArcSec}[a+b\,x]\,\text{Sin}\left[\frac{1}{2}\,\text{ArcSec}[a+b\,x]\right]+3\,a\,\text{ArcSec}[a+b\,x]^2\,\text{Sin}\left[\frac{1}{2}\,\text{ArcSec}[a+b\,x]\right]+$$
$$\left.\left.6\,a^3\,\text{ArcSec}[a+b\,x]^2\,\text{Sin}\left[\frac{1}{2}\,\text{ArcSec}[a+b\,x]\right]\right)\right)\Big/$$

$$\left.\left(6\,(a+b\,x)^3\left(-1+\frac{a}{a+b\,x}\right)^3\left(\text{Cos}\left[\frac{1}{2}\,\text{ArcSec}[a+b\,x]\right]-\text{Sin}\left[\frac{1}{2}\,\text{ArcSec}[a+b\,x]\right]\right)\right)\right)$$

Problem 31: Result more than twice size of optimal antiderivative.

$$\int\frac{\text{ArcSec}[a+b\,x]^2}{x}\,dx$$

Optimal (type 4, 310 leaves, 17 steps):

$$\text{ArcSec}[a+b x]^2 \text{Log}\left[1-\frac{a e^{i \text{ArcSec}[a+b x]}}{1-\sqrt{1-a^2}}\right]+\text{ArcSec}[a+b x]^2 \text{Log}\left[1-\frac{a e^{i \text{ArcSec}[a+b x]}}{1+\sqrt{1-a^2}}\right]-$$
$$\text{ArcSec}[a+b x]^2 \text{Log}\left[1+e^{2 i \text{ArcSec}[a+b x]}\right]-2 i \text{ArcSec}[a+b x] \text{PolyLog}\left[2,\frac{a e^{i \text{ArcSec}[a+b x]}}{1-\sqrt{1-a^2}}\right]-$$
$$2 i \text{ArcSec}[a+b x] \text{PolyLog}\left[2,\frac{a e^{i \text{ArcSec}[a+b x]}}{1+\sqrt{1-a^2}}\right]+i \text{ArcSec}[a+b x] \text{PolyLog}\left[2,-e^{2 i \text{ArcSec}[a+b x]}\right]+$$
$$2 \text{PolyLog}\left[3,\frac{a e^{i \text{ArcSec}[a+b x]}}{1-\sqrt{1-a^2}}\right]+2 \text{PolyLog}\left[3,\frac{a e^{i \text{ArcSec}[a+b x]}}{1+\sqrt{1-a^2}}\right]-\frac{1}{2} \text{PolyLog}\left[3,-e^{2 i \text{ArcSec}[a+b x]}\right]$$

Result (type 4, 813 leaves) :

$$\text{ArcSec}[a+b x]^2 \text{Log}\left[1+\frac{a e^{i \text{ArcSec}[a+b x]}}{-1+\sqrt{1-a^2}}\right]+\text{ArcSec}[a+b x]^2 \text{Log}\left[1+\frac{\left(-1+\sqrt{1-a^2}\right) e^{i \text{ArcSec}[a+b x]}}{a}\right]-$$

$$4 \text{ArcSec}[a+b x] \text{ArcSin}\left[\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right] \text{Log}\left[1+\frac{\left(-1+\sqrt{1-a^2}\right) e^{i \text{ArcSec}[a+b x]}}{a}\right]+$$

$$\text{ArcSec}[a+b x]^2 \text{Log}\left[1-\frac{a e^{i \text{ArcSec}[a+b x]}}{1+\sqrt{1-a^2}}\right]+\text{ArcSec}[a+b x]^2 \text{Log}\left[1-\frac{\left(1+\sqrt{1-a^2}\right) e^{i \text{ArcSec}[a+b x]}}{a}\right]+$$

$$4 \text{ArcSec}[a+b x] \text{ArcSin}\left[\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right] \text{Log}\left[1-\frac{\left(1+\sqrt{1-a^2}\right) e^{i \text{ArcSec}[a+b x]}}{a}\right]-$$

$$2 \text{ArcSec}[a+b x]^2 \text{Log}\left[1+e^{2 i \text{ArcSec}[a+b x]}\right]+\text{ArcSec}[a+b x]^2 \text{Log}\left[\frac{2\left(\frac{1}{a+b x}+i \sqrt{1-\frac{1}{(a+b x)^2}}\right)}{a+b x}\right]-$$

$$\text{ArcSec}[a+b x]^2 \text{Log}\left[1+\frac{\left(-1+\sqrt{1-a^2}\right)\left(\frac{1}{a+b x}+i \sqrt{1-\frac{1}{(a+b x)^2}}\right)}{a}\right]+$$

$$4 \text{ArcSec}[a+b x] \text{ArcSin}\left[\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right] \text{Log}\left[1+\frac{\left(-1+\sqrt{1-a^2}\right)\left(\frac{1}{a+b x}+i \sqrt{1-\frac{1}{(a+b x)^2}}\right)}{a}\right]-$$

$$\text{ArcSec}[a+b x]^2 \text{Log}\left[1-\frac{\left(1+\sqrt{1-a^2}\right)\left(\frac{1}{a+b x}+i \sqrt{1-\frac{1}{(a+b x)^2}}\right)}{a}\right]-$$

$$4 \text{ArcSec}[a+b x] \text{ArcSin}\left[\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right] \text{Log}\left[1-\frac{\left(1+\sqrt{1-a^2}\right)\left(\frac{1}{a+b x}+i \sqrt{1-\frac{1}{(a+b x)^2}}\right)}{a}\right]-$$

$$2 i \text{ArcSec}[a+b x] \text{PolyLog}\left[2,-\frac{a e^{i \text{ArcSec}[a+b x]}}{-1+\sqrt{1-a^2}}\right]-2 i \text{ArcSec}[a+b x] \text{PolyLog}\left[2,\frac{a e^{i \text{ArcSec}[a+b x]}}{1+\sqrt{1-a^2}}\right]+$$

$$i \text{ArcSec}[a+b x] \text{PolyLog}\left[2,-e^{2 i \text{ArcSec}[a+b x]}\right]+2 \text{PolyLog}\left[3,-\frac{a e^{i \text{ArcSec}[a+b x]}}{-1+\sqrt{1-a^2}}\right]+$$

$$2 \text{PolyLog}\left[3,\frac{a e^{i \text{ArcSec}[a+b x]}}{1+\sqrt{1-a^2}}\right]-\frac{1}{2} \text{PolyLog}\left[3,-e^{2 i \text{ArcSec}[a+b x]}\right]$$

Problem 32: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcSec}[a+b x]^2}{x^2} \, dx$$

Optimal (type 4, 244 leaves, 12 steps):

$$-\frac{b\,\text{ArcSec}[a+b\,x]^2}{a}-\frac{\text{ArcSec}[a+b\,x]^2}{x}-\frac{2\,i\,b\,\text{ArcSec}[a+b\,x]\,\text{Log}\left[1-\frac{a\,e^{i\,\text{ArcSec}[a+b\,x]}}{1-\sqrt{1-a^2}}\right]}{a\sqrt{1-a^2}}+\frac{2\,i\,b\,\text{ArcSec}[a+b\,x]\,\text{Log}\left[1-\frac{a\,e^{i\,\text{ArcSec}[a+b\,x]}}{1+\sqrt{1-a^2}}\right]}{a\sqrt{1-a^2}}-\frac{2\,b\,\text{PolyLog}\left[2,\frac{a\,e^{i\,\text{ArcSec}[a+b\,x]}}{1-\sqrt{1-a^2}}\right]}{a\sqrt{1-a^2}}+\frac{2\,b\,\text{PolyLog}\left[2,\frac{a\,e^{i\,\text{ArcSec}[a+b\,x]}}{1+\sqrt{1-a^2}}\right]}{a\sqrt{1-a^2}}$$

Result (type 4, 686 leaves):

$$-\frac{1}{a}\left(\frac{(a+b\,x)\,\text{ArcSec}[a+b\,x]^2}{x}+\frac{1}{\sqrt{-1+a^2}}2\,b\left(2\,\text{ArcSec}[a+b\,x]\,\text{ArcTanh}\left[\frac{(-1+a)\,\text{Cot}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]}{\sqrt{-1+a^2}}\right]-2\,\text{ArcCos}\left[\frac{1}{a}\right]\text{ArcTanh}\left[\frac{(1+a)\,\text{Tan}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]}{\sqrt{-1+a^2}}\right]+\left(\text{ArcCos}\left[\frac{1}{a}\right]-2\,i\,\text{ArcTanh}\left[\frac{(-1+a)\,\text{Cot}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]}{\sqrt{-1+a^2}}\right]+2\,i\,\text{ArcTanh}\left[\frac{(1+a)\,\text{Tan}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]}{\sqrt{-1+a^2}}\right]\right)\text{Log}\left[\frac{\sqrt{-1+a^2}\;e^{-\frac{1}{2}i\,\text{ArcSec}[a+b\,x]}}{\sqrt{2}\,\sqrt{a}\,\sqrt{-\frac{b\,x}{a+b\,x}}}\right]+\left(\text{ArcCos}\left[\frac{1}{a}\right]+2\,i\left(\text{ArcTanh}\left[\frac{(-1+a)\,\text{Cot}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]}{\sqrt{-1+a^2}}\right]-\text{ArcTanh}\left[\frac{(1+a)\,\text{Tan}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]}{\sqrt{-1+a^2}}\right]\right)\right)\text{Log}\left[\frac{\sqrt{-1+a^2}\;e^{\frac{1}{2}i\,\text{ArcSec}[a+b\,x]}}{\sqrt{2}\,\sqrt{a}\,\sqrt{-\frac{b\,x}{a+b\,x}}}\right]-\left(\text{ArcCos}\left[\frac{1}{a}\right]-2\,i\,\text{ArcTanh}\left[\frac{(1+a)\,\text{Tan}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]}{\sqrt{-1+a^2}}\right]\right)\text{Log}\left[\left((-1+a)\left(i+i\,a+\sqrt{-1+a^2}\right)\left(-i+\text{Tan}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]\right)\right)\Big/\left(a\left(-1+a+\sqrt{-1+a^2}\;\text{Tan}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]\right)\right)\right]-\right.$$

$$\left(\text{ArcCos}\left[\frac{1}{a}\right]+2\,i\,\text{ArcTanh}\left[\frac{(1+a)\,\text{Tan}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]}{\sqrt{-1+a^2}}\right]\right)$$
$$\text{Log}\left[\left((-1+a)\left(-i-i\,a+\sqrt{-1+a^2}\right)\left(i+\text{Tan}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]\right)\right)\Big/\left(a\left(-1+a+\sqrt{-1+a^2}\,\text{Tan}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]\right)\right)\right]+$$
$$i\left(-\text{PolyLog}\left[2,\left(\left(1-i\sqrt{-1+a^2}\right)\left(1-a+\sqrt{-1+a^2}\,\text{Tan}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]\right)\right)\Big/\left(a\left(-1+a+\sqrt{-1+a^2}\,\text{Tan}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]\right)\right)\right]+\right.$$
$$\left.\left.\left.\text{PolyLog}\left[2,\left(\left(1+i\sqrt{-1+a^2}\right)\left(1-a+\sqrt{-1+a^2}\,\text{Tan}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]\right)\right)\Big/\left(a\left(-1+a+\sqrt{-1+a^2}\,\text{Tan}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]\right)\right)\right]\right)\right)\right)$$

Problem 33: Unable to integrate problem.

$$\int x^2\,\text{ArcSec}[a+b\,x]^3\,dx$$

Optimal (type 4, 494 leaves, 25 steps):

$$\frac{(a+b x) \text{ArcSec}[a+b x]}{b^3}-\frac{3 i a \text{ArcSec}[a+b x]^2}{b^3}+$$
$$\frac{3 a (a+b x) \sqrt{1-\frac{1}{(a+b x)^2}} \text{ArcSec}[a+b x]^2}{b^3}-\frac{(a+b x)^2 \sqrt{1-\frac{1}{(a+b x)^2}} \text{ArcSec}[a+b x]^2}{2 b^3}+$$
$$\frac{a^3 \text{ArcSec}[a+b x]^3}{3 b^3}+\frac{1}{3} x^3 \text{ArcSec}[a+b x]^3+\frac{i \text{ArcSec}[a+b x]^2 \text{ArcTan}\left[e^{i \text{ArcSec}[a+b x]}\right]}{b^3}+$$
$$\frac{6 i a^2 \text{ArcSec}[a+b x]^2 \text{ArcTan}\left[e^{i \text{ArcSec}[a+b x]}\right]}{b^3}-\frac{\text{ArcTanh}\left[\sqrt{1-\frac{1}{(a+b x)^2}}\right]}{b^3}+$$
$$\frac{6 a \text{ArcSec}[a+b x] \text{Log}\left[1+e^{2 i \text{ArcSec}[a+b x]}\right]}{b^3}-\frac{i \text{ArcSec}[a+b x] \text{PolyLog}\left[2,-i e^{i \text{ArcSec}[a+b x]}\right]}{b^3}-$$
$$\frac{6 i a^2 \text{ArcSec}[a+b x] \text{PolyLog}\left[2,-i e^{i \text{ArcSec}[a+b x]}\right]}{b^3}+$$
$$\frac{i \text{ArcSec}[a+b x] \text{PolyLog}\left[2,i e^{i \text{ArcSec}[a+b x]}\right]}{b^3}+\frac{6 i a^2 \text{ArcSec}[a+b x] \text{PolyLog}\left[2,i e^{i \text{ArcSec}[a+b x]}\right]}{b^3}-$$
$$\frac{3 i a \text{PolyLog}\left[2,-e^{2 i \text{ArcSec}[a+b x]}\right]}{b^3}+\frac{\text{PolyLog}\left[3,-i e^{i \text{ArcSec}[a+b x]}\right]}{b^3}+$$
$$\frac{6 a^2 \text{PolyLog}\left[3,-i e^{i \text{ArcSec}[a+b x]}\right]}{b^3}-\frac{\text{PolyLog}\left[3,i e^{i \text{ArcSec}[a+b x]}\right]}{b^3}-\frac{6 a^2 \text{PolyLog}\left[3,i e^{i \text{ArcSec}[a+b x]}\right]}{b^3}$$

Result (type 8, 14 leaves) :

$$\int x^2 \text{ArcSec}[a+b x]^3 \, dx$$

Problem 36: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcSec}[a+b x]^3}{x} \, dx$$

Optimal (type 4, 430 leaves, 20 steps) :

$$\text{ArcSec}[a+b x]^3 \text{Log}\left[1-\frac{a e^{i \text{ArcSec}[a+b x]}}{1-\sqrt{1-a^2}}\right]+\text{ArcSec}[a+b x]^3 \text{Log}\left[1-\frac{a e^{i \text{ArcSec}[a+b x]}}{1+\sqrt{1-a^2}}\right]-\text{ArcSec}[a+b x]^3 \text{Log}\left[1+e^{2 i \text{ArcSec}[a+b x]}\right]-3 i \text{ArcSec}[a+b x]^2 \text{PolyLog}\left[2,\frac{a e^{i \text{ArcSec}[a+b x]}}{1-\sqrt{1-a^2}}\right]-3 i \text{ArcSec}[a+b x]^2 \text{PolyLog}\left[2,\frac{a e^{i \text{ArcSec}[a+b x]}}{1+\sqrt{1-a^2}}\right]+\frac{3}{2} i \text{ArcSec}[a+b x]^2 \text{PolyLog}\left[2,-e^{2 i \text{ArcSec}[a+b x]}\right]+6 \text{ArcSec}[a+b x] \text{PolyLog}\left[3,\frac{a e^{i \text{ArcSec}[a+b x]}}{1-\sqrt{1-a^2}}\right]+6 \text{ArcSec}[a+b x] \text{PolyLog}\left[3,\frac{a e^{i \text{ArcSec}[a+b x]}}{1+\sqrt{1-a^2}}\right]-\frac{3}{2} \text{ArcSec}[a+b x] \text{PolyLog}\left[3,-e^{2 i \text{ArcSec}[a+b x]}\right]+6 i \text{PolyLog}\left[4,\frac{a e^{i \text{ArcSec}[a+b x]}}{1-\sqrt{1-a^2}}\right]+6 i \text{PolyLog}\left[4,\frac{a e^{i \text{ArcSec}[a+b x]}}{1+\sqrt{1-a^2}}\right]-\frac{3}{4} i \text{PolyLog}\left[4,-e^{2 i \text{ArcSec}[a+b x]}\right]$$

Result (type 4, 1058 leaves):

$$2 \text{ArcSec}[a+b x]^3 \text{Log}\left[1+\frac{a e^{i \text{ArcSec}[a+b x]}}{-1+\sqrt{1-a^2}}\right]+\text{ArcSec}[a+b x]^3 \text{Log}\left[1+\frac{\left(-1+\sqrt{1-a^2}\right) e^{i \text{ArcSec}[a+b x]}}{a}\right]-6 \text{ArcSec}[a+b x]^2 \text{ArcSin}\left[\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right] \text{Log}\left[1+\frac{\left(-1+\sqrt{1-a^2}\right) e^{i \text{ArcSec}[a+b x]}}{a}\right]+2 \text{ArcSec}[a+b x]^3 \text{Log}\left[1-\frac{a e^{i \text{ArcSec}[a+b x]}}{1+\sqrt{1-a^2}}\right]+\text{ArcSec}[a+b x]^3 \text{Log}\left[1-\frac{\left(1+\sqrt{1-a^2}\right) e^{i \text{ArcSec}[a+b x]}}{a}\right]+6 \text{ArcSec}[a+b x]^2 \text{ArcSin}\left[\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right] \text{Log}\left[1-\frac{\left(1+\sqrt{1-a^2}\right) e^{i \text{ArcSec}[a+b x]}}{a}\right]-3 \text{ArcSec}[a+b x]^3 \text{Log}\left[1+e^{2 i \text{ArcSec}[a+b x]}\right]+2 \text{ArcSec}[a+b x]^3 \text{Log}\left[\frac{2\left(\frac{1}{a+b x}+i \sqrt{1-\frac{1}{(a+b x)^2}}\right)}{a+b x}\right]-\text{ArcSec}[a+b x]^3 \text{Log}\left[1+\frac{a\left(\frac{1}{a+b x}+i \sqrt{1-\frac{1}{(a+b x)^2}}\right)}{-1+\sqrt{1-a^2}}\right]-\text{ArcSec}[a+b x]^3 \text{Log}\left[1+\frac{\left(-1+\sqrt{1-a^2}\right)\left(\frac{1}{a+b x}+i \sqrt{1-\frac{1}{(a+b x)^2}}\right)}{a}\right]+6 \text{ArcSec}[a+b x]^2 \text{ArcSin}\left[\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right] \text{Log}\left[1+\frac{\left(-1+\sqrt{1-a^2}\right)\left(\frac{1}{a+b x}+i \sqrt{1-\frac{1}{(a+b x)^2}}\right)}{a}\right]-$$

$$\text{ArcSec}[a+b x]^3 \text{Log}\left[1-\frac{a\left(\frac{1}{a+b x}+i \sqrt{1-\frac{1}{(a+b x)^2}}\right)}{1+\sqrt{1-a^2}}\right]-$$
$$\text{ArcSec}[a+b x]^3 \text{Log}\left[1-\frac{\left(1+\sqrt{1-a^2}\right)\left(\frac{1}{a+b x}+i \sqrt{1-\frac{1}{(a+b x)^2}}\right)}{a}\right]-$$
$$6 \text{ArcSec}[a+b x]^2 \text{ArcSin}\left[\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right] \text{Log}\left[1-\frac{\left(1+\sqrt{1-a^2}\right)\left(\frac{1}{a+b x}+i \sqrt{1-\frac{1}{(a+b x)^2}}\right)}{a}\right]-$$
$$3 i \text{ArcSec}[a+b x]^2 \text{PolyLog}\left[2,-\frac{a e^{i \text{ArcSec}[a+b x]}}{-1+\sqrt{1-a^2}}\right]-$$
$$3 i \text{ArcSec}[a+b x]^2 \text{PolyLog}\left[2,\frac{a e^{i \text{ArcSec}[a+b x]}}{1+\sqrt{1-a^2}}\right]+$$
$$\frac{3}{2} i \text{ArcSec}[a+b x]^2 \text{PolyLog}\left[2,-e^{2 i \text{ArcSec}[a+b x]}\right]+$$
$$6 \text{ArcSec}[a+b x] \text{PolyLog}\left[3,-\frac{a e^{i \text{ArcSec}[a+b x]}}{-1+\sqrt{1-a^2}}\right]+6 \text{ArcSec}[a+b x] \text{PolyLog}\left[3,\frac{a e^{i \text{ArcSec}[a+b x]}}{1+\sqrt{1-a^2}}\right]-$$
$$\frac{3}{2} \text{ArcSec}[a+b x] \text{PolyLog}\left[3,-e^{2 i \text{ArcSec}[a+b x]}\right]+6 i \text{PolyLog}\left[4,-\frac{a e^{i \text{ArcSec}[a+b x]}}{-1+\sqrt{1-a^2}}\right]+$$
$$6 i \text{PolyLog}\left[4,\frac{a e^{i \text{ArcSec}[a+b x]}}{1+\sqrt{1-a^2}}\right]-\frac{3}{4} i \text{PolyLog}\left[4,-e^{2 i \text{ArcSec}[a+b x]}\right]$$

Problem 37: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcSec}[a+b x]^3}{x^2} \, dx$$

Optimal (type 4, 362 leaves, 14 steps):

$$-\frac{b \text{ArcSec}[a+b x]^3}{a}-\frac{\text{ArcSec}[a+b x]^3}{x}-$$
$$\frac{3 i b \text{ArcSec}[a+b x]^2 \text{Log}\left[1-\frac{a e^{i \text{ArcSec}[a+b x]}}{1-\sqrt{1-a^2}}\right]}{a \sqrt{1-a^2}}+\frac{3 i b \text{ArcSec}[a+b x]^2 \text{Log}\left[1-\frac{a e^{i \text{ArcSec}[a+b x]}}{1+\sqrt{1-a^2}}\right]}{a \sqrt{1-a^2}}-$$
$$\frac{6 b \text{ArcSec}[a+b x] \text{PolyLog}\left[2,\frac{a e^{i \text{ArcSec}[a+b x]}}{1-\sqrt{1-a^2}}\right]}{a \sqrt{1-a^2}}+\frac{6 b \text{ArcSec}[a+b x] \text{PolyLog}\left[2,\frac{a e^{i \text{ArcSec}[a+b x]}}{1+\sqrt{1-a^2}}\right]}{a \sqrt{1-a^2}}-$$
$$\frac{6 i b \text{PolyLog}\left[3,\frac{a e^{i \text{ArcSec}[a+b x]}}{1-\sqrt{1-a^2}}\right]}{a \sqrt{1-a^2}}+\frac{6 i b \text{PolyLog}\left[3,\frac{a e^{i \text{ArcSec}[a+b x]}}{1+\sqrt{1-a^2}}\right]}{a \sqrt{1-a^2}}$$

Result (type 4, 1664 leaves):

$$-\frac{1}{a\sqrt{-1+a^2}\,x}$$

$$\Bigg(a\sqrt{-1+a^2}\,\text{ArcSec}[a+b\,x]^3+\sqrt{-1+a^2}\,b\,x\,\text{ArcSec}[a+b\,x]^3+6\,b\,x\,\text{ArcCos}\left[-\frac{1}{a}\right]\text{ArcSec}[a+b\,x]$$

$$\text{Log}\left[\frac{\sqrt{-1+a^2}\,e^{-\text{ArcTanh}\left[\frac{(1+a)\,\text{Tan}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]}{\sqrt{-1+a^2}}\right]}}{\sqrt{2}\,\sqrt{a}\,\sqrt{1+a\,\text{Cosh}\left[2\,\text{ArcTanh}\left[\frac{(1+a)\,\text{Tan}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]}{\sqrt{-1+a^2}}\right]\right]}}\right]+$$

$$12\,b\,x\,\text{ArcSec}[a+b\,x]\,\text{ArcTan}\left[\text{Cot}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]\right]$$

$$\text{Log}\left[\frac{\sqrt{-1+a^2}\,e^{-\text{ArcTanh}\left[\frac{(1+a)\,\text{Tan}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]}{\sqrt{-1+a^2}}\right]}}{\sqrt{2}\,\sqrt{a}\,\sqrt{1+a\,\text{Cosh}\left[2\,\text{ArcTanh}\left[\frac{(1+a)\,\text{Tan}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]}{\sqrt{-1+a^2}}\right]\right]}}\right]+$$

$$12\,b\,x\,\text{ArcSec}[a+b\,x]\,\text{ArcTan}\left[\text{Tan}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]\right]$$

$$\text{Log}\left[\frac{\sqrt{-1+a^2}\,e^{-\text{ArcTanh}\left[\frac{(1+a)\,\text{Tan}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]}{\sqrt{-1+a^2}}\right]}}{\sqrt{2}\,\sqrt{a}\,\sqrt{1+a\,\text{Cosh}\left[2\,\text{ArcTanh}\left[\frac{(1+a)\,\text{Tan}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]}{\sqrt{-1+a^2}}\right]\right]}}\right]+6\,b\,x\,\text{ArcCos}\left[-\frac{1}{a}\right]$$

$$\text{ArcSec}[a+b\,x]\,\text{Log}\left[\frac{\sqrt{-1+a^2}\,e^{\text{ArcTanh}\left[\frac{(1+a)\,\text{Tan}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]}{\sqrt{-1+a^2}}\right]}}{\sqrt{2}\,\sqrt{a}\,\sqrt{1+a\,\text{Cosh}\left[2\,\text{ArcTanh}\left[\frac{(1+a)\,\text{Tan}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]}{\sqrt{-1+a^2}}\right]\right]}}\right]-$$

$$12\,b\,x\,\text{ArcSec}[a+b\,x]\,\text{ArcTan}\left[\text{Cot}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]\right]$$

$$\text{Log}\left[\frac{\sqrt{-1+a^2}\,e^{\text{ArcTanh}\left[\frac{(1+a)\,\text{Tan}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]}{\sqrt{-1+a^2}}\right]}}{\sqrt{2}\,\sqrt{a}\,\sqrt{1+a\,\text{Cosh}\left[2\,\text{ArcTanh}\left[\frac{(1+a)\,\text{Tan}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]}{\sqrt{-1+a^2}}\right]\right]}}\right]-$$

$$12\,b\,x\,\text{ArcSec}[a+b\,x]\,\text{ArcTan}\left[\text{Tan}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]\right]$$

$$\text{Log}\Big[\frac{\sqrt{-1+a^2}\ e^{\text{ArcTanh}\left[\frac{(1+a)\ \text{Tan}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]}{\sqrt{-1+a^2}}\right]}}{\sqrt{2}\ \sqrt{a}\ \sqrt{1+a\,\text{Cosh}\left[2\,\text{ArcTanh}\left[\frac{(1+a)\ \text{Tan}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]}{\sqrt{-1+a^2}}\right]\right]}}\Big]-$$

$$6\,b\,x\,\text{ArcCos}\left[-\frac{1}{a}\right]\text{ArcSec}[a+b\,x]\ \text{Log}\Big[\left(\left(-1+a^2\right)\sqrt{-\frac{b\,x}{(-1+a)\ (1+a+b\,x)}}\right)\Big/$$

$$\left(\sqrt{a}\ \sqrt{-\frac{\left(-1+a^2\right)\ (a+b\,x)}{b\,x}}\ \left(\sqrt{-1+a^2}+(1+a)\ \text{Tan}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]\right)\right)\Big]-12\,b\,x$$

$$\text{ArcSec}[a+b\,x]\ \text{ArcTan}\left[\text{Cot}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]\right]\ \text{Log}\Big[\left(\left(-1+a^2\right)\sqrt{-\frac{b\,x}{(-1+a)\ (1+a+b\,x)}}\right)\Big/$$

$$\left(\sqrt{a}\ \sqrt{-\frac{\left(-1+a^2\right)\ (a+b\,x)}{b\,x}}\ \left(\sqrt{-1+a^2}+(1+a)\ \text{Tan}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]\right)\right)\Big]-12\,b\,x$$

$$\text{ArcSec}[a+b\,x]\ \text{ArcTan}\left[\text{Tan}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]\right]\ \text{Log}\Big[\left(\left(-1+a^2\right)\sqrt{-\frac{b\,x}{(-1+a)\ (1+a+b\,x)}}\right)\Big/$$

$$\left(\sqrt{a}\ \sqrt{-\frac{\left(-1+a^2\right)\ (a+b\,x)}{b\,x}}\ \left(\sqrt{-1+a^2}+(1+a)\ \text{Tan}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]\right)\right)\Big]-$$

$$6\,b\,x\,\text{ArcCos}\left[-\frac{1}{a}\right]\text{ArcSec}[a+b\,x]\ \text{Log}\Big[\frac{\sqrt{-1+a^2}+(1+a)\ \text{Tan}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]}{2\sqrt{a}\ \sqrt{-\frac{\left(-1+a^2\right)\ (a+b\,x)}{b\,x}}\ \sqrt{-\frac{b\,x}{(-1+a)\ (1+a+b\,x)}}}\Big]+$$

$$12\,b\,x\,\text{ArcSec}[a+b\,x]\ \text{ArcTan}\left[\text{Cot}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]\right]$$

$$\text{Log}\Big[\frac{\sqrt{-1+a^2}+(1+a)\ \text{Tan}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]}{2\sqrt{a}\ \sqrt{-\frac{\left(-1+a^2\right)\ (a+b\,x)}{b\,x}}\ \sqrt{-\frac{b\,x}{(-1+a)\ (1+a+b\,x)}}}\Big]+12\,b\,x\,\text{ArcSec}[a+b\,x]$$

$$\text{ArcTan}\left[\text{Tan}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]\right]\ \text{Log}\Big[\frac{\sqrt{-1+a^2}+(1+a)\ \text{Tan}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]}{2\sqrt{a}\ \sqrt{-\frac{\left(-1+a^2\right)\ (a+b\,x)}{b\,x}}\ \sqrt{-\frac{b\,x}{(-1+a)\ (1+a+b\,x)}}}\Big]-$$

$$3\,b\,x\,\text{ArcSec}[a+b\,x]^2\ \text{Log}\Big[-\frac{\left(-1+a-i\sqrt{-1+a^2}\right)\left(-1+\frac{(1+a)\ \text{Tan}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]}{\sqrt{-1+a^2}}\right)}{a+i\,a\,\text{Tan}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]}\Big]+$$

$$3\,b\,x\,\text{ArcSec}[a+b\,x]^2\,\text{Log}\left[\frac{\left(-1+a+i\sqrt{-1+a^2}\right)\left(1+\frac{(1+a)\,\text{Tan}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]}{\sqrt{-1+a^2}}\right)}{a+i\,a\,\text{Tan}\left[\frac{1}{2}\text{ArcSec}[a+b\,x]\right]}\right]+$$

$$6\,i\,b\,x\,\text{ArcSec}[a+b\,x]\,\text{PolyLog}\left[2,\frac{\left(1-i\sqrt{-1+a^2}\right)\left(\frac{1}{a+b\,x}-i\sqrt{1-\frac{1}{(a+b\,x)^2}}\right)}{a}\right]-$$

$$6\,i\,b\,x\,\text{ArcSec}[a+b\,x]\,\text{PolyLog}\left[2,\frac{\left(1+i\sqrt{-1+a^2}\right)\left(\frac{1}{a+b\,x}-i\sqrt{1-\frac{1}{(a+b\,x)^2}}\right)}{a}\right]+$$

$$6\,b\,x\,\text{PolyLog}\left[3,\frac{\left(1-i\sqrt{-1+a^2}\right)\left(\frac{1}{a+b\,x}-i\sqrt{1-\frac{1}{(a+b\,x)^2}}\right)}{a}\right]-$$

$$\left.6\,b\,x\,\text{PolyLog}\left[3,\frac{\left(1+i\sqrt{-1+a^2}\right)\left(\frac{1}{a+b\,x}-i\sqrt{1-\frac{1}{(a+b\,x)^2}}\right)}{a}\right]\right)$$

Problem 38: Result more than twice size of optimal antiderivative.

$$\int x\left(a+b\,\text{ArcSec}\left[c+d\,x^2\right]\right)dx$$

Optimal (type 3, 58 leaves, 7 steps):

$$\frac{a\,x^2}{2}+\frac{b\left(c+d\,x^2\right)\text{ArcSec}\left[c+d\,x^2\right]}{2\,d}-\frac{b\,\text{ArcTanh}\left[\sqrt{1-\frac{1}{\left(c+d\,x^2\right)^2}}\right]}{2\,d}$$

Result (type 3, 154 leaves):

$$\frac{a\,x^2}{2}+\frac{1}{2}b\,x^2\,\text{ArcSec}\left[c+d\,x^2\right]-$$

$$\left(b\left(c+d\,x^2\right)\sqrt{\frac{-1+c^2+2\,c\,d\,x^2+d^2\,x^4}{\left(c+d\,x^2\right)^2}}\left(c\,\text{ArcTan}\left[\frac{1}{\sqrt{-1+c^2+2\,c\,d\,x^2+d^2\,x^4}}\right]+\right.\right.$$

$$\left.\left.\text{Log}\left[c+d\,x^2+\sqrt{-1+c^2+2\,c\,d\,x^2+d^2\,x^4}\right]\right)\right)\Big/\left(2\,d\sqrt{-1+c^2+2\,c\,d\,x^2+d^2\,x^4}\right)$$

Problem 39: Result more than twice size of optimal antiderivative.

$$\int x^2 \left(a + b\,\text{ArcSec}\left[c + d\,x^3\right]\right) \, dx$$

Optimal (type 3, 58 leaves, 7 steps):

$$\frac{a\,x^3}{3} + \frac{b\,\left(c + d\,x^3\right)\,\text{ArcSec}\left[c + d\,x^3\right]}{3\,d} - \frac{b\,\text{ArcTanh}\left[\sqrt{1 - \frac{1}{\left(c+d\,x^3\right)^2}}\right]}{3\,d}$$

Result (type 3, 154 leaves):

$$\frac{a\,x^3}{3} + \frac{1}{3}\,b\,x^3\,\text{ArcSec}\left[c + d\,x^3\right] - \left(b\,\left(c + d\,x^3\right)\,\sqrt{\frac{-1 + c^2 + 2\,c\,d\,x^3 + d^2\,x^6}{\left(c + d\,x^3\right)^2}}\,\left(c\,\text{ArcTan}\left[\frac{1}{\sqrt{-1 + c^2 + 2\,c\,d\,x^3 + d^2\,x^6}}\right] + \text{Log}\left[c + d\,x^3 + \sqrt{-1 + c^2 + 2\,c\,d\,x^3 + d^2\,x^6}\,\right]\right)\right) \Big/ \left(3\,d\,\sqrt{-1 + c^2 + 2\,c\,d\,x^3 + d^2\,x^6}\right)$$

Problem 40: Result more than twice size of optimal antiderivative.

$$\int x^3 \left(a + b\,\text{ArcSec}\left[c + d\,x^4\right]\right) \, dx$$

Optimal (type 3, 58 leaves, 7 steps):

$$\frac{a\,x^4}{4} + \frac{b\,\left(c + d\,x^4\right)\,\text{ArcSec}\left[c + d\,x^4\right]}{4\,d} - \frac{b\,\text{ArcTanh}\left[\sqrt{1 - \frac{1}{\left(c+d\,x^4\right)^2}}\right]}{4\,d}$$

Result (type 3, 137 leaves):

$$\frac{a\,x^4}{4} + \frac{b\,\left(c + d\,x^4\right)\,\text{ArcSec}\left[c + d\,x^4\right]}{4\,d} - \left(b\,\sqrt{-1 + \left(c + d\,x^4\right)^2}\,\left(-\text{Log}\left[1 - \frac{c + d\,x^4}{\sqrt{-1 + \left(c + d\,x^4\right)^2}}\right] + \text{Log}\left[1 + \frac{c + d\,x^4}{\sqrt{-1 + \left(c + d\,x^4\right)^2}}\right]\right)\right) \Big/ \left(8\,d\,\left(c + d\,x^4\right)\,\sqrt{1 - \frac{1}{\left(c + d\,x^4\right)^2}}\right)$$

Problem 41: Result more than twice size of optimal antiderivative.

$$\int x^{-1+n}\,\text{ArcSec}\left[a + b\,x^n\right] \, dx$$

Optimal (type 3, 49 leaves, 6 steps):

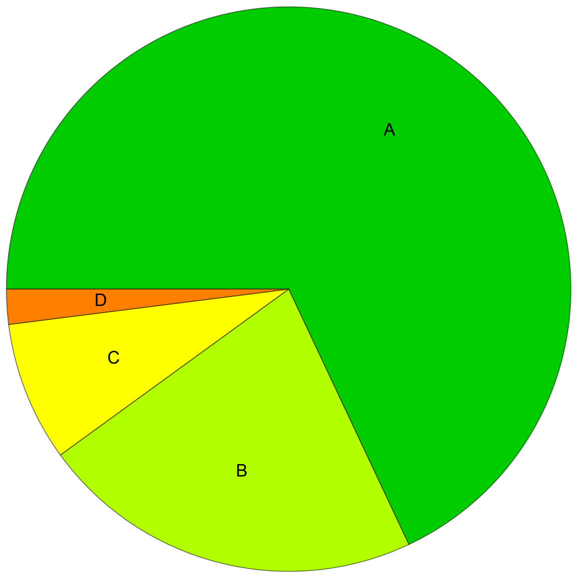
$$\frac{(a+b\,x^n)\,\text{ArcSec}[a+b\,x^n]}{b\,n}-\frac{\text{ArcTanh}\left[\sqrt{1-\frac{1}{(a+b\,x^n)^2}}\right]}{b\,n}$$

Result (type 3, 130 leaves) :

$$\frac{(a+b\,x^n)\,\text{ArcSec}[a+b\,x^n]}{b\,n}-\left(\sqrt{-1+(a+b\,x^n)^2}\left(-\text{Log}\left[1-\frac{a+b\,x^n}{\sqrt{-1+(a+b\,x^n)^2}}\right]+\text{Log}\left[1+\frac{a+b\,x^n}{\sqrt{-1+(a+b\,x^n)^2}}\right]\right)\right)\Big/\left(2\,b\,n\,(a+b\,x^n)\sqrt{1-\frac{1}{(a+b\,x^n)^2}}\right)$$

Summary of Integration Test Results

50 integration problems



A - 34 optimal antiderivatives

B - 11 more than twice size of optimal antiderivatives

C - 4 unnecessarily complex antiderivatives

D - 1 unable to integrate problems

E - 0 integration timeouts